An additional difficulty students often face is in recognizing that a given sum is actually a sum of a geometric sequence and in properly identifying a and b. The practice problems in this section as well as the homework problems should give you some practice.

The following identity is also true, although we will not use this often in this class.

$$\sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2}, \quad |r| < 1 \tag{1.25}$$

1.6 Practice Problems

1. Compute $5 + \frac{10}{3} + \frac{20}{9} + \frac{40}{27} + \ldots$

2. Compute $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \ldots$

3. Simplify $\sum_{n=2}^{9} 2^{3n} 3^{-2n}$

4. Compute $\sum_{n=2}^{\infty} 2^{3n} 3^{-2n}$

5. Compute $\sum_{n=-\infty}^{-2} 2^{-3n} 3^{2n}$. See if you can substitute $m = -n$ and obtain the expression in the previous problem

6. Simply $\sum_{n=2}^{\infty} x^n 3^{-n}$ an expression as a rational function of x. Evaluate this function for $x = 2$

7. Compute $\sum_{n=1}^{\infty} \cos^n(\pi t)$ and express as a function of t

8. Compute $\sum_{n=1}^{\infty} \cos(n\pi t)$ and express as a function of t. Hint: Use Euler's formula to convert this sum of two geometric series.

9. Simplify $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n e^{j\omega n}$

10. Compute $e^{j\frac{\pi}{2}} + \frac{1}{2}e^{j\pi} + \frac{1}{4}e^{j\frac{3\pi}{2}} + \ldots + \frac{1}{2^9}e^{j\frac{10\pi}{2}}$. Simplify your answer into a complex number in Cartesian form

11. Prove the result in (1.23) and (1.24)

12. For any two given integers k and N, what is $\sum_{n=0}^{N-1} e^{\frac{j2\pi kn}{N}}$?

13. Just for intellectual curiosity - Can you prove the results in (1.21) and (1.25)?