1.4 Practice Problems

1. Express these numbers in Cartesian form

   a) $2e^{j\pi/4}$

   b) $(1+j)(2-3j)$

   c) $e^{j\pi/4} + e^{j3\pi/4}$

   d) $\frac{2}{e^j}$

   e) $e^{j\pi/3} + e^{-j\pi/3}$

2. Express these numbers in polar form

   a) $1+j$,

   b) $\frac{1}{2} + j\frac{\sqrt{3}}{2}$, $\frac{1}{2} - j\frac{\sqrt{3}}{2}$, $-\frac{1}{2} + j\frac{\sqrt{3}}{2}$, $-\frac{1}{2} - j\frac{\sqrt{3}}{2}$

   c) $(1-j)\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$

   d) $\frac{1+j}{2j}$

   e) $e^{j\pi/4} + e^{j3\pi/4}$

   f) $1 + e^j$

3. Compute the magnitude and phase of the following complex numbers. Do not explicitly compute the numbers in Cartesian form unless it is required. Try to compute the magnitude and phase using what you know about the magnitude and phase of products of complex numbers.

   a) $(1-j)\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$

   b) $e^{j\pi/2}(1+j)(1+3j)$

   c) $je^{j\pi/3}$

   d) $e^{j\pi/4} + e^{j3\pi/4}$

   e) $(1+3j)^2$

   f) $(1-3j)/(1+3j)^2$

   g) $e^{j\pi/5} \times e^{j2\pi/5} \times e^{j3\pi/5} \ldots e^{j9\pi/5}$

   h) $e^{j\pi/5} \times e^{j2\pi/5} \times e^{j3\pi/5} \ldots e^{j9\pi/5} \times e^{j10\pi/5}$

4. Let $z_1 = 1, z_2 = -\frac{1}{2} + j\frac{\sqrt{3}}{2}, z_3 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$

   a) What are z_1^3, z_2^3 and z_3^3?

   b) Show that $z_3 = z_2^2$

   c) Show that $z_1 + z_2 + z_3 = 0$

   Can you now see why z_1, z_2, z_3 can be called the cube roots of unity. They are usually expressed as $1, \omega, \omega^2$. Part c shows that the sum of the cube roots of unity is zero. In one of the homework problems, we will show that this true for nth roots of unity for any n.

5. Let $z_1 = 2e^{j\pi/4}$ and $z_2 = 8e^{j\pi/3}$. Find and express your answer in Cartesian and polar form

   a) $2z_1 - z_2$

   b) $\frac{1}{z_1}$

   c) $\frac{z_1}{z_2^2}$

   d) $\sqrt[3]{z_2}$

6. Let z be any complex number. Is it true that $(e^z)^\star = e^{z^\star}$?

7. Prove that

$$\int e^{ax} \cos(bx)\, dx = \frac{e^{ax}}{a^2 + b^2} \left(a\cos(bx) + b\sin(bx)\right)$$

   You can use integration tricks you learned in your calculus class to solve this problem. That is not the point of the exercise. Try using Euler's identity and then using integration of exponentials to see if you can solve the problem.

8. Plot the magnitude and phase of the function $X(f) = e^{j\pi f} + e^{j5\pi f}$, for $-1 \leq f \leq 1$.

9. Just for fun - what is j^j?

1.4.1 References

A good online reference for complex numbers is the wiki page `http://en.wikipedia.org/wiki/Complex_number`.