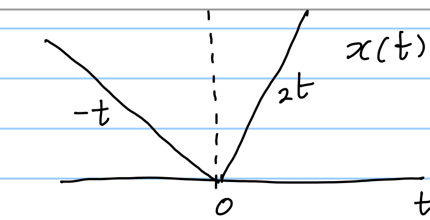
Transformations of signals defined piecewise

Note Title 2/1/2014

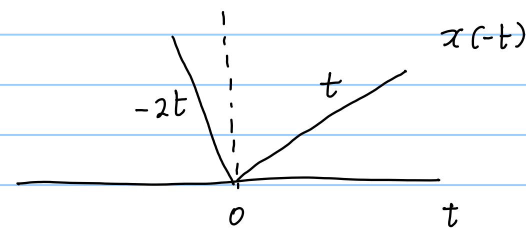
Example:

$$\boxed{\begin{array}{lll} x(t) & = 2t & t>0 \\ \rightarrow & = -t & t<0 \\ & = 0 & t=0 \end{array}}$$



Find $x(-t)$ and $\frac{1}{2}\left[x(t)+x(-t)\right]$

$$\begin{array}{lll} x(-t) & = 2\cdot(-t) & \\ & = -(-t) & \\ & = 0 & \end{array} \qquad \boxed{\begin{array}{l} -t>0 \\ -t<0 \\ -t=0 \end{array}}$$



$$\boxed{\begin{array}{lll} x(-t) \rightarrow & = -2t & t<0 \\ & = t & t>0 \\ & = 0 & t=0 \end{array}}$$

$$\begin{array}{lll} \frac{1}{2}\left[x(t)+x(-t)\right] & = -\frac{3}{2}t & t<0 \\ & = \frac{3t}{2} & t>0 \\ & = 0 & t=0 \end{array}$$

Suppose we want to find $x(1-2t)$

$$x(t) = \begin{cases} 2t & t>0 \\ -t & t<0 \\ 0 & t=0 \end{cases}$$

$$x(1-2t) = \begin{cases} 2\,(1-2t) & 1-2t>0 \Rightarrow t<\frac{1}{2} \\ -1\cdot(1-2t) & 1-2t<0 \Rightarrow t>1/2 \\ 0 & 1-2t=0 \Rightarrow t=1/2 \end{cases}$$

Generalization

Suppose

$$\begin{aligned} x(t) &= g_1(t) && h_1(t)>0 \\ &= g_2(t) && h_2(t)>0 \\ &\vdots \\ &= g_N(t) && h_N(t)>0 \\ &= g_{N+1}(t) && h_{N+1}(t)=0 \\ &\vdots \\ &= g_{N+M}(t) && h_{N+M}(t)=0 \end{aligned}$$

what is $x(f(t))$?

$$
\begin{aligned}
x(f(t)) &= g_1(f(t)) && h_1(f(t)) > 0 \\
&= g_2(f(t)) && h_2(f(t)) > 0 \\
&\vdots \\
&= g_N(f(t)) && h_N(f(t)) > 0 \\
&= g_{N+1}(f(t)) && h_{N+1}(f(t)) = 0 \\
&\vdots \\
&= g_{N+M}(f(t)) && h_{N+M}(f(t)) = 0
\end{aligned}
$$