Properties of the CTFT

Note Title 7/13/2012

* Linearity $x(t) \longleftrightarrow X(j\omega)$, $y(t) \longleftrightarrow Y(j\omega)$

$$z(t) = a\,x(t) + b\,y(t) \longleftrightarrow a\,X(j\omega) + b\,Y(j\omega)$$

* Time shifting $x(t) \longleftrightarrow X(j\omega)$

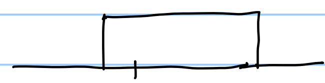
$$z(t) = x(t - t_0) \longleftrightarrow e^{-j\omega t_0} X(j\omega)$$

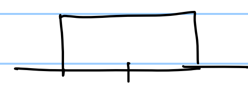
$$Z(j\omega) = \int x(t - t_0)\, e^{-j\omega t}\, dt \qquad \text{sub } \tau = t - t_0$$

$$= \int x(\tau)\, e^{-j\omega(\tau + t_0)}\, dt = e^{-j\omega t_0} X(j\omega)$$

Mag remains unchanged , phase is different

Example



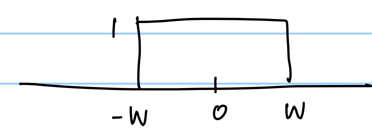
start with $x(t) =$  $\longleftrightarrow 2T_0 \,\text{sinc}\left(\frac{\omega T_0}{\pi}\right) = \frac{2}{\omega} \sin(\omega T_0)$

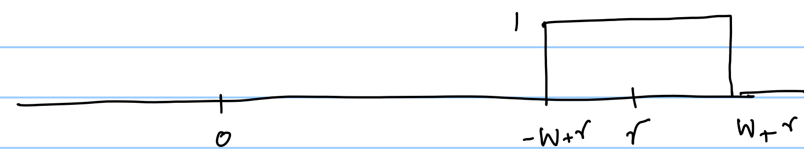
$$Z(j\omega) = e^{-j\omega T_1} \, 2T_0 \,\text{sinc}\left(\frac{\omega T_0}{\pi}\right)$$

Shift in Frequency (Modulation property)

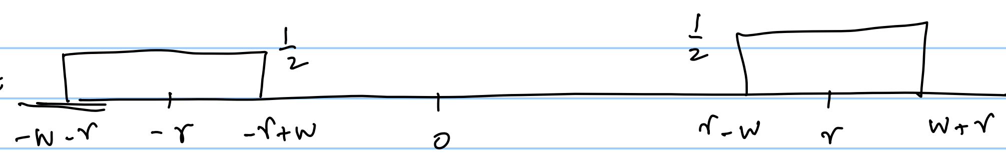
$$x(t) \longleftrightarrow X(j\omega)$$

$$e^{j\Omega t} x(t) \longleftrightarrow X(j(\omega - \Omega))$$

Example $\frac{1}{\pi t} \sin Wt \longleftrightarrow$ 

$z(t) = e^{j\gamma t} \cdot \frac{1}{\pi t} \sin Wt$ 

$\gamma = 900$ MHz

$z(t) = \cos \gamma t \cdot \frac{1}{\pi t} \sin Wt$ 

Time and Frequency Scaling

$$x(t) \longleftrightarrow X(j\omega)$$

$$x(at) \longleftrightarrow \frac{1}{|a|} X\left(j\frac{\omega}{a}\right)$$ a is a real constant

Special case, $a = -1$

$$x(-t) \longleftrightarrow X(-j\omega)$$

Conjugation and Symmetry

$$x(t) \longleftrightarrow X(j\omega)$$

$$x^*(t) \longleftrightarrow X^*(-j\omega)$$

If $x(t)$ is real

$$x(t) = x^*(t)$$

$$\Rightarrow \boxed{\begin{aligned} X(j\omega) &= X^*(-j\omega) \\ X(-j\omega) &= X^*(j\omega) \end{aligned}}$$

Conjugate Symmetry

<u>Proof:</u>

$$x(t) = \frac{1}{2\pi} \int X(j\omega)\, e^{j\omega t}\, d\omega$$

$$x^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(j\omega)\, e^{-j\omega t}\, d\omega$$

sub $r = -\omega \qquad dr = -d\omega$

$$x^*(t) = \frac{1}{2\pi} - \int_{+\infty}^{-\infty} X^*(-jr)\, e^{jrt}\, dr$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(-jr)\, e^{jrt}\, dr$$

Example $x(t) = e^{-at} u(t) \longleftrightarrow X(j\omega) = \frac{1}{a+j\omega}$

$X^*(j\omega) = \frac{1}{a-j\omega}$ $X(-j\omega) = \frac{1}{a-j\omega}$

Same

If $x(t)$ is strictly imaginary $x(t) = -x^*(t)$

$X(j\omega) = -X^*(-j\omega) \Rightarrow -X(j\omega) = X^*(-j\omega)$ ← Conjugate anti-symmetry

Suppose $x(t)$ has some symmetry, i.e

Conjugate symmetric $\quad x(t) = x^*(-t)$

$$x(-t) = x^*(t)$$