

Multiplication - Convolution Duality

Note Title

10/25/2011

Fill up proof

Examples

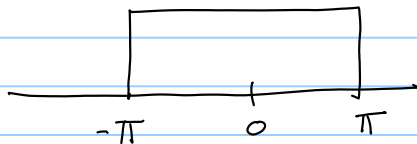
$$1) \quad x(t) = \frac{1}{\pi t} \sin \pi t$$

$$h(t) = \frac{1}{\pi t} \sin 2\pi t$$

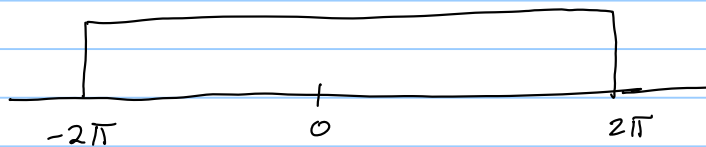
$$\text{Find } y(t) = x(t) * h(t)$$

We will find $Y(j\omega) = X(j\omega) H(j\omega)$ and then find $y(t)$

$$X(j\omega) =$$



$$H(j\omega) =$$



$$Y(j\omega) = X(j\omega) H(j\omega) = X(j\omega) \Rightarrow y(t) = x(t)$$

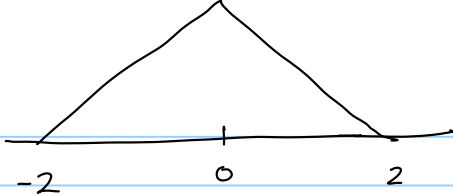
Ex. 2 (3.32 from Haykin)

$$\text{Find } x(t) \text{ where } X(j\omega) = \frac{4}{\omega^2} \sin^2 \omega$$

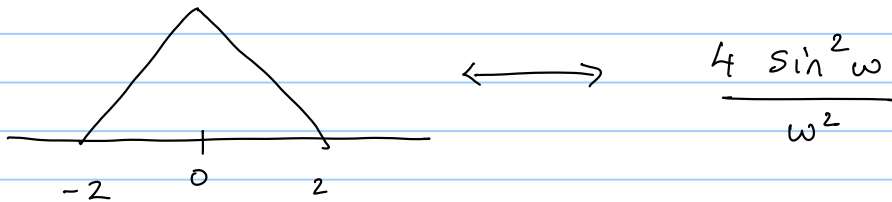
$$X(j\omega) = \left(\frac{2}{\omega} \sin \omega \right) \left(\frac{2}{\omega} \sin \omega \right)$$

$Y(j\omega) \qquad Y(j\omega)$

$$y(t) =$$

$$x(t) = y(t) * y(t) = \text{triangle}(t)$$


Final answer

$$\text{triangle}(t) \longleftrightarrow \frac{4 \sin^2 \omega}{\omega^2}$$


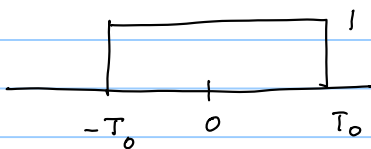
Example 3 : Multiplication in time

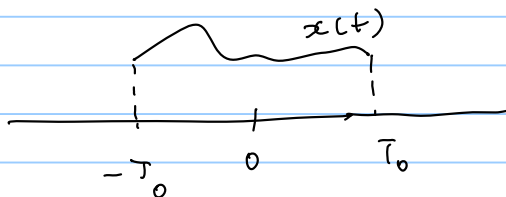
Windowing :

Suppose we have $x(t) \longleftrightarrow X(j\omega)$

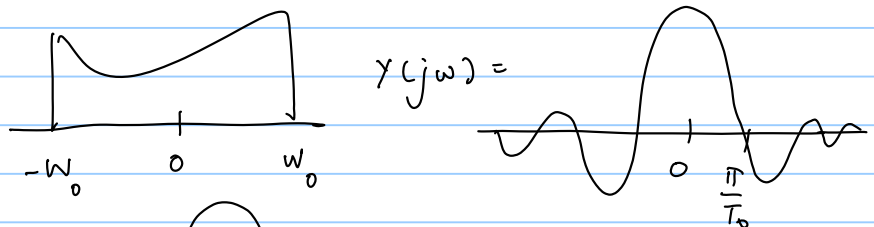
If $X(j\omega) = 0$ for $|\omega| > W_0$

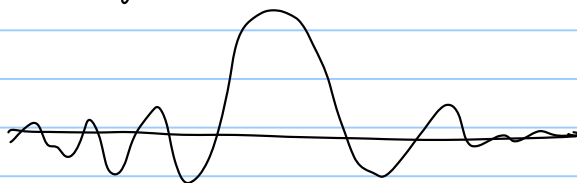
We say that $x(t)$ is bandlimited

$$\text{say } y(t) = \text{rect}(t)$$


$$z(t) = x(t) y(t) = \text{windowed } x(t)$$


$$Z(j\omega) = \frac{1}{2\pi} X(j\omega) * Y(j\omega)$$

$$X(j\omega) = \text{bandlimited signal} \quad Y(j\omega) = \text{sinc function}$$


$$Z(j\omega) = \text{rippled bandlimited signal}$$


Any timelimited Signal Cannot be Bandlimited