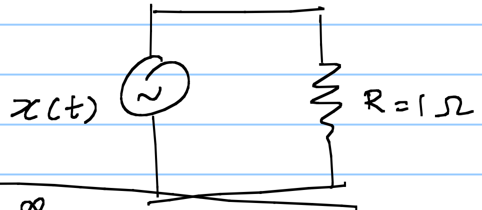
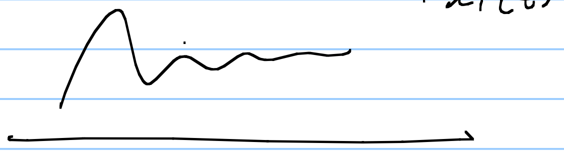
Signal Energy and Signal Power

real signal $x(t)$

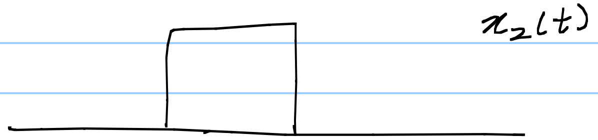


Power $P(t) = x^2(t)$

$$\boxed{E_x = \text{Energy} = \int_{-\infty}^{\infty} x^2(t)\, dt}$$



How big a Signal is ?

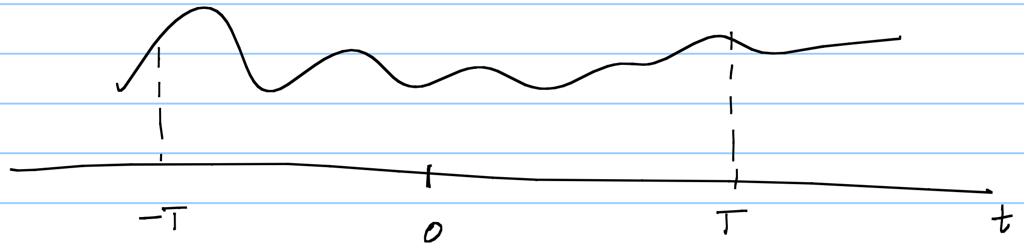


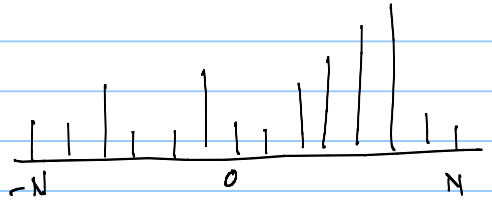
Suppose $x(t)$ is a Complex Signal

$$\boxed{E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt}$$

If $x[n]$ is a discrete-time signal, $E_x = \sum_{-\infty}^{\infty} |x[n]|^2$

Power:





CT

$$\boxed{P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt}$$

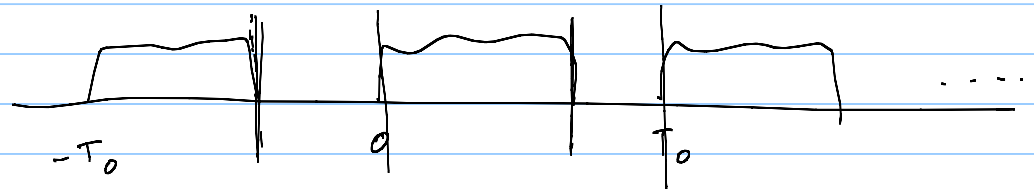
DT

$$\boxed{P_x = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2}$$

Power of a Periodic Signal

CT

$x(t)$ is periodic with time period T_0



$$P_x = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |x(t)|^2 \, dt$$

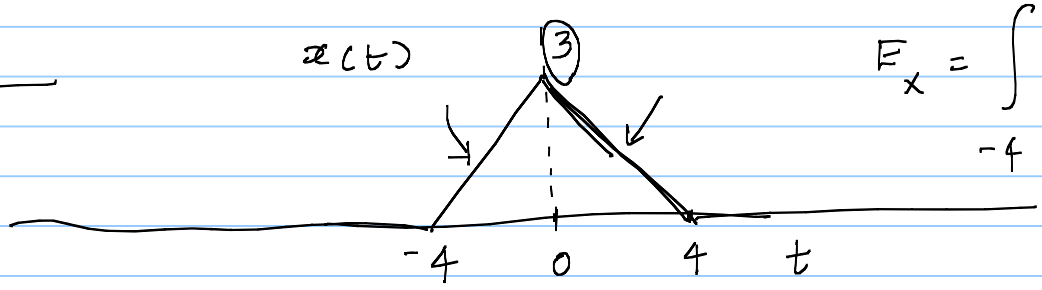
DT

$$P_x = \frac{1}{N_0} \sum_{n_0}^{n_0+N_0-1} |x[n]|^2$$

$x(t)$ is an energy signal if $0 < E_x < \infty$

$x(t)$ is a power signal if $0 < P_x < \infty$

Example



$$E_x = \int_{-4}^{4} |x(t)|^2 \, dt = \int_{-4}^{4} x^2(t) \, dt$$

$$\boxed{\begin{aligned} x(t) &= 3\left(1 - \frac{t}{4}\right) & 0 \leq t \leq 4 \\ &= 3\left(1 + \frac{t}{4}\right) & -4 \leq t \leq 0 \\ &= 0 & \text{otherwise} \end{aligned}}$$

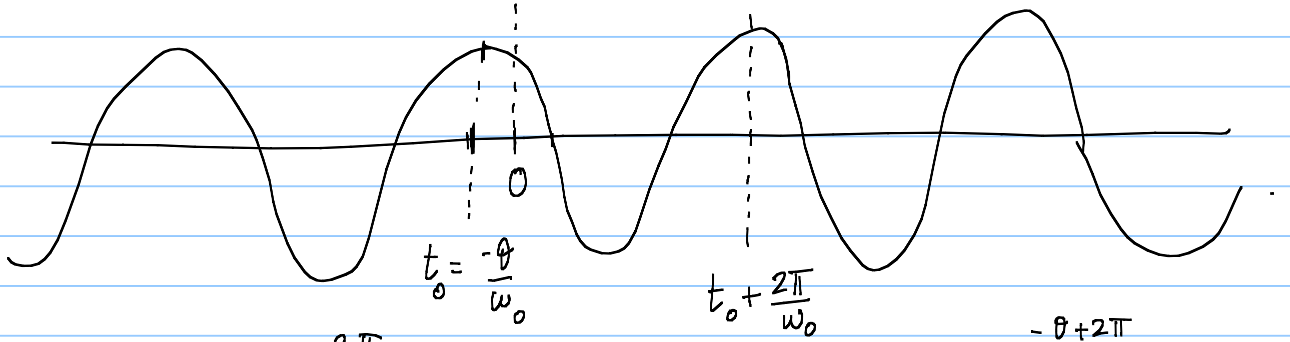
$$\int_{-4}^{4} x^2(t)\,dt = 9\int_{-4}^{0} \left(1+\frac{t}{4}\right)^2 dt + 9\int_{0}^{4}\left(1-\frac{t}{4}\right)^2 dt$$

$$= 9\left[\frac{\left(1+t/4\right)^3}{\frac{3}{4}}\right]_{-4}^{0} + 9\left[\frac{\left(1-t/4\right)^3}{\frac{3}{4}}\right]_{0}^{4}$$

$$= 9.\frac{4}{3} + 9.\frac{4}{3} = 24$$

$x(t)$ is an Energy Signal !

$$x(t) = A\cos(\omega_0 t + \overset{\swarrow}{\theta})$$



E_x is unbounded!

$$T_0 = \frac{2\pi}{\omega_0}$$

$$P_x = \frac{1}{T_0}\int_0^{\frac{2\pi}{\omega_0}} A^2 \cos^2(\omega_0 t + \theta)\; dt \quad = \frac{1}{T_0}\int_{-\frac{\theta}{\omega_0}}^{\frac{-\theta+2\pi}{\omega_0}} A^2 \cos^2(\omega_0 t + \theta)\; dt$$

$$P_x = \frac{A^2}{T_0}\int_{-\theta/\omega_0}^{\frac{2\pi-\theta}{\omega_0}} \frac{1+\cos(2\omega_0 t + 2\theta)}{2}\; dt$$

$$= \frac{A^2}{T_0}\left[\frac{t}{\textcircled{2}} + \cancelto{0}{\frac{\sin(2\omega_0 t + 2\theta)}{4\omega_0}}\right]_{-\frac{\theta}{\omega_0}}^{\frac{2\pi-\theta}{\omega_0}}$$

$$\sin(4\pi - \cancel{2\theta} + \cancel{2\theta}) - \sin(0)$$

$$= \frac{A^2}{T_0}\left[\frac{1}{2}\left(\frac{2\pi-\cancel{\theta}}{\omega_0} - \frac{-\cancel{\theta}}{\omega_0}\right) + \cancelto{0}{\frac{\sin\left(2\cdot\cancel{\omega_0}\cdot\frac{2\pi-\theta}{\cancel{\omega_0}}+2\theta\right) - \sin\left(2\cancel{\omega_0}\cdot\frac{-\theta}{\cancel{\omega_0}}+2\theta\right)}{4\omega_0}}\right]$$

$$= \frac{1}{2}\times\frac{A^2}{\cancel{T_0}}\times\cancel{\frac{2\pi}{\omega_0}} \qquad T_0 = \frac{2\pi}{\omega_0} \qquad = \frac{A^2}{2}\ !$$

$x(t)$ is a Power Signal

$x(t) = e^{j\omega_0 t}$ $\quad T_0 = \frac{2\pi}{\omega_0}$ $\quad e^{j\omega_0 \left(t + \frac{2\pi}{\omega_0}\right)} = e^{j(\omega_0 t + 2\pi)}$

$$e^{j\theta} = e^{j(\theta + 2\pi)}$$

$x(t)$ is periodic woth a period of $\frac{2\pi}{\omega_0}$

for all θ

$$|e^{j\theta}| = 1$$

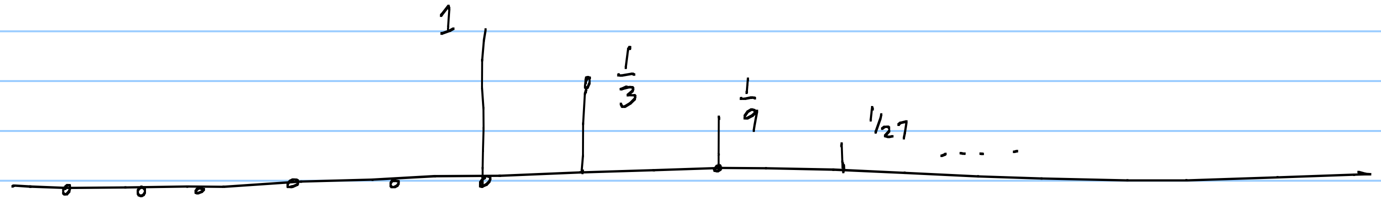
$$P_x = \frac{1}{T_0} \int_0^{T_0} \boxed{\left| e^{j\omega_0 t} \right|^2} dt = \frac{1}{T_0} \int_0^{T_0} 1 .\, dt$$

$$|e^{j\omega_0 t}| = 1$$

$$= 1 \Big|_0$$

what is the Energy of $x[n] = \left(\frac{1}{3}\right)^n \quad n \geq 0$

$= 0 \quad$ otherwise



$$E_x = \sum_{-\infty}^{\infty} x^2[n] = \sum_{0}^{\infty} x^2[n] = \sum_{0}^{\infty} \left(\left(\frac{1}{3}\right)^n\right)^2 \swarrow$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n \swarrow$$

$$= \frac{1}{1-\frac{1}{9}} = \frac{9}{8} \, !$$