Note Title 6/10/2012

Even and Odd Signals

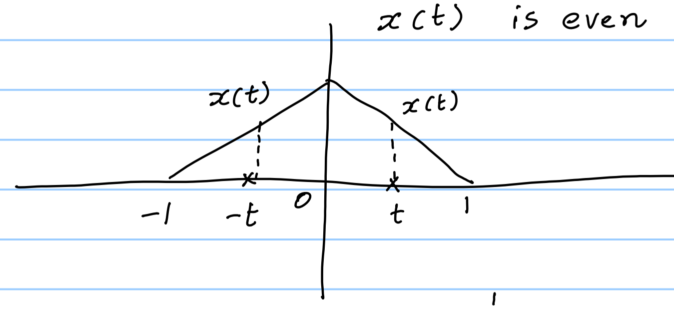
A signal $x(t)$ is said to be

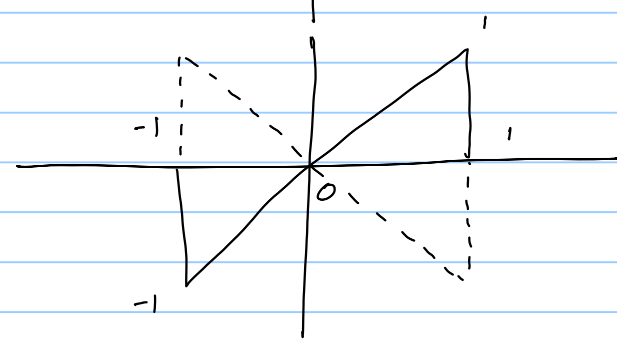
Even if $\boxed{x(t) = x(-t)}$

odd if $\boxed{x(t) = -x(-t)}$

$\boxed{x(-t) = -x(t)}$

$x(t)$ is even





Any signal $x(t)$ can be written as the sum of an even signal and an odd signal

$$x(t) = \underbrace{x_e(t)}_{\text{Even part}} + \underbrace{x_o(t)}_{\text{Odd part}}$$

$$x(t) = \underbrace{\left(\frac{1}{2}x(t) + \frac{1}{2}x(-t)\right)}_{\text{Even part}} + \underbrace{\left(\frac{1}{2}x(t) - \frac{1}{2}x(-t)\right)}_{\text{odd part}}$$

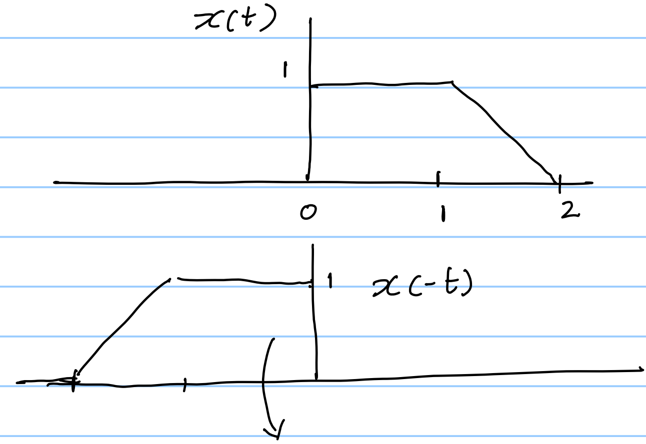
$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] \Rightarrow x_e(-t) = \frac{1}{2}\left[x(-t) + x(t)\right]$$

$$x_e(t) = x_e(-t) \Rightarrow x_e(t) \text{ is even}$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right] \Rightarrow x_o(-t) = \frac{1}{2}\left[x(-t) - x(t)\right] = -x_o(t)$$

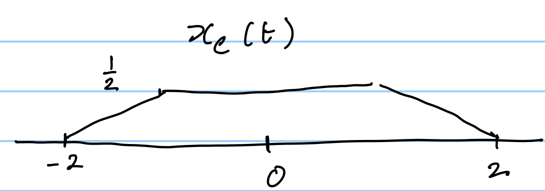
hence $x_o(t)$ is odd.

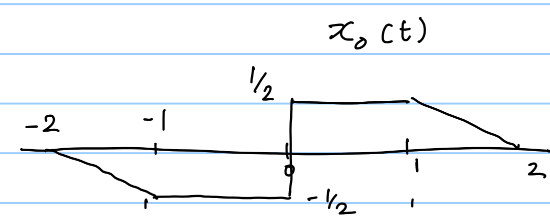
<u>Example:</u> Find the even and odd parts of



$$x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right] \checkmark$$

$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right] \checkmark$$





Find the even and odd parts of

$$x(t) = \cos t + \overbrace{\sin t + \cos t \sin t}$$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t)\sin(-t)$$

$$= \cos t - \sin t - \cos t \sin t$$

$$x_e(t) = \cos t \qquad x_o(t) = \sin t + \cos t \sin t$$

Discrete-time Signals

$x[n]$ is even if $\quad x[n] = x[-n]$

$x[n]$ is odd if $\quad x[n] = -x[-n]$

Properties of Even and Odd Signals

Even Signal $\pm$ Even Signal = Even Signal

odd Signal $\pm$ odd Signal = odd Signal

Even signal $\pm$ odd signal = We cant say anything

$$\begin{array}{ccccc} x_{e_3}(t) = & x_{e_1}(t) & + & x_{e_2}(t) \\ \| & \| & & \| \\ x_{e_3}(-t) = & x_{e_1}(-t) & + & x_{e_2}(-t) \end{array}$$

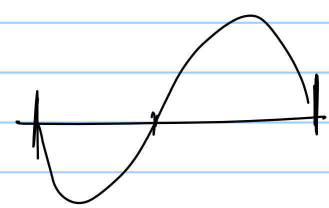
Even $\times$ Even = Even

odd $\times$ odd = Even

Even $\times$ odd = odd

Integrals

$x(t)$ is odd. what is $\displaystyle\int_{-A}^{A} x(t)\,dt = 0$



$$\int_{-1}^{1} \underbrace{\sin^3 t}\,dt = 0$$

$x(t)$ is even. $\displaystyle\int_{-A}^{A} x(t)\,dt = 2\int_{0}^{A} x(t)\,dt$

Conjugate Symmetry

Suppose $x(t)$ is a complex signal

$$x(t) = a(t) + j\,b(t) = r(t)\,e^{j\theta(t)}$$

Even signal – Conjugate Symmetric Signal

If $x(t) = x^*(-t)$

Conjugate anti-symmetric if $x(t) = -x^*(-t) \Rightarrow x^*(t) = -x(-t)$

$$-x(t) = -x^*(-t)$$

If $x(t)$ is real $\longrightarrow$ $X(j\omega)$ is conjugate symmetric

$x(t)$ is real and <u>even</u> $\longrightarrow$ $X(j\omega)$ is <u>real</u>

$\parallel$

$x(t)$ is complex, Conjugate Symmetric $\longrightarrow$ $X(j\omega)$ is <u>real</u>

Conjugate Symmetry: $x(t) = x^*(-t)$

$$(a(t) + jb(t) = a(-t) - j\,b(-t)$$

$$a(t) = a(-t) \Longrightarrow a(t) \text{ is even}$$
$$b(t) = -b(-t) \Longrightarrow b(t) \text{ is odd}$$

$$x(t) = x^*(-t)$$
$$r(t)\,e^{j\theta(t)} = r(-t)\,e^{-j\theta(-t)}$$

$$r(t) = r(-t) \Longrightarrow r(t) \text{ is even}$$

$$\theta(t) = -\theta(-t) \Longrightarrow \theta(t) \text{ is odd}$$