Note Title 10/25/2011

Differentiation and Integration Properties

$$\boxed{\begin{array}{ccc} \frac{d}{dt}x(t) & \longleftrightarrow & j\omega\, X(j\omega) \\ -jt\,x(t) & \longleftrightarrow & \frac{d}{d\omega}X(j\omega) \end{array}}$$

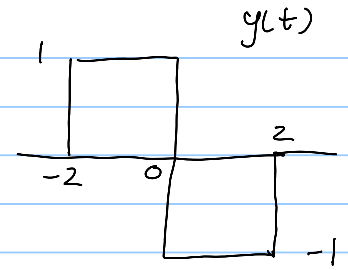
Proof:

$$x(t) = \frac{1}{2\pi}\int X(j\omega)\, e^{j\omega t}\, d\omega$$

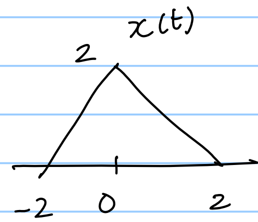
$$\frac{d}{dt}x(t) = \frac{1}{2\pi}\int \underbrace{j\omega\, X(j\omega)}\, e^{j\omega t}\, d\omega$$

$$\Rightarrow \quad \frac{d}{dt}x(t) \longleftrightarrow j\omega\, X(j\omega)$$

Example

Find the F.T. of 

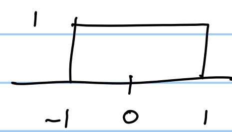
using

F.T. of 

$= \dfrac{4 \sin^2 \omega}{\omega^2}$

Notice that $y(t) = \dfrac{d}{dt} y(t) \Rightarrow X(j\omega) = \dfrac{j\omega\, 4 \sin^2 \omega}{\omega^2}$

Can we check another way?

Start with 

$\dfrac{2 \sin \omega}{\omega}$

same

$X(j\omega) = \left(e^{j\omega} - e^{-j\omega}\right) \dfrac{2 \sin \omega}{\omega} = 4j \dfrac{\sin^2 \omega}{\omega}$

We will see yet another way to compute this FT using the integration property

Total Area under the curve property

E.g. what is $I = \int_{-\infty}^{\infty} \frac{1}{\pi t} \sin Wt \; dt$?

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$\Rightarrow \quad X(j0) = \int_{-\infty}^{\infty} x(t)\, dt \qquad x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, d\omega$$

$I = X(j0)$ where $X(j\omega) =$