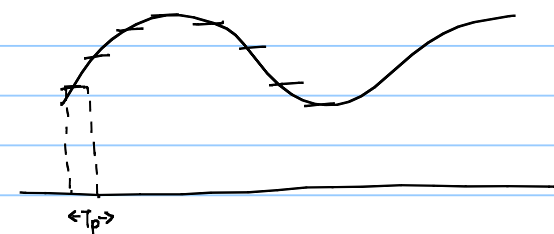
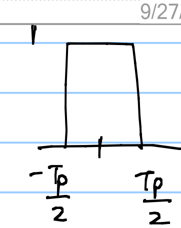
Deriving the expression for CT Convolution

Note Title 9/27/2011

Consider an arbitrary input $x(t)$



$\mathrm{rect}(t) =$



$$x(t) \approx \cdots + x(-T_p)\,\mathrm{rect}\left(\frac{t+T_p}{T_p}\right) + x(0)\,\mathrm{rect}(t) + x(T_p)\,\mathrm{rect}\left(\frac{t-T_p}{T_p}\right) + \cdots\cdots$$

<u>Not</u> the unit rectangle

$$= \sum_{n=-\infty}^{\infty} x(nT_p)\,\mathrm{rect}\left(\frac{t-nT_p}{T_p}\right) = \sum T_p\, x(nT_p)\left[\frac{1}{T_p}\,\mathrm{rect}\left(\frac{t-nT_p}{T_p}\right)\right]$$

Suppose h_p is the response of the system to an input $\frac{1}{T_p}\,\mathrm{rect}\left(\frac{t-nT_p}{T_p}\right)$

Then $y(t)$ corresponding to $x(t)$ is $y(t) = \sum T_p\, x(nT)\; h_p(t-nT_p)$

Taking limit $T_p \to 0$ $\quad \frac{1}{T_p} \text{rect}\left(\frac{t}{T_p}\right) \longrightarrow \delta(t)$, let response be $h(t)$ (impulse)

Now, look at $y(t) = \sum T_p \; x(nT_p) \; h_p(t - nT_p)$ and let $\boxed{\tau = nT_p}$

$$= \int d\tau \; x(\tau) \; h(t-\tau)$$

Therefore, $\boxed{y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau}$

Notice that $\boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau}$

we already know this from the sifting property