Discrete-Time Convolution

Let $h[n]$ be the impulse response of a linear and time-invariant (LTI) system. If the signal $x[n]$ is input to the system, the output signal from the system is given by $y[n] = \sum_k x[k]\ h[n-k]$. This operation is called convolution and we say that the signal $y[n]$ is the convolution of the signal $x[n]$ and the signal $h[n]$ and we denote this by $y[n] = x[n] \star h[n]$. Thus,

$$\boxed{y[n] = x[n] \star h[n] = \sum_k x[k]\ h[n-k]}$$

To compute the signal $y[n]$, perform the following steps.

Step 1: Think of $x[n]$ and $h[n]$ as signals $x[k]$ and $h[k]$ respectively, i.e., with the independent variable being k instead of n.

Step 2: Flip $h[k]$ about the Y-axis to obtain the signal $h[-k]$.

Step 3: To compute the signal $y[n]$ for a fixed value of n, shift the signal $h[-k]$ by n units to the right to obtain the signal $h[n-k]$. When n is negative, this amounts to shifting the signal $h[-k]$ to the left, but mathematically it is equivalent to shift right by a negative number.

Step 4: Compute $w_n[k] = x[k]\ h[n-k]$, i.e., $w_n[k]$ is the product of the signals $x[k]$ and $h[n-k]$.

Step 5: Compute $y[n] = \sum_k w_n[k] = \sum_k x[k]\ h[n-k]$ by summing the values of $w_n[k]$ for all values of k.

This will give you the value of the signal $y[n]$ for one value of n. Repeat this procedure for every integer value of n, i.e., $n \in [\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots]$ to obtain the full signal $y[n]$. In practice, it will be easy to start with large negative values of n and increase n.