Computing the intermediate signal in DT convolution

Note Title 6/18/2011

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Discrete-time Convolution

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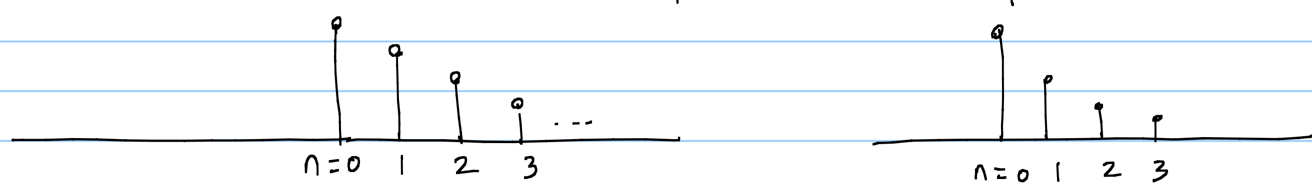
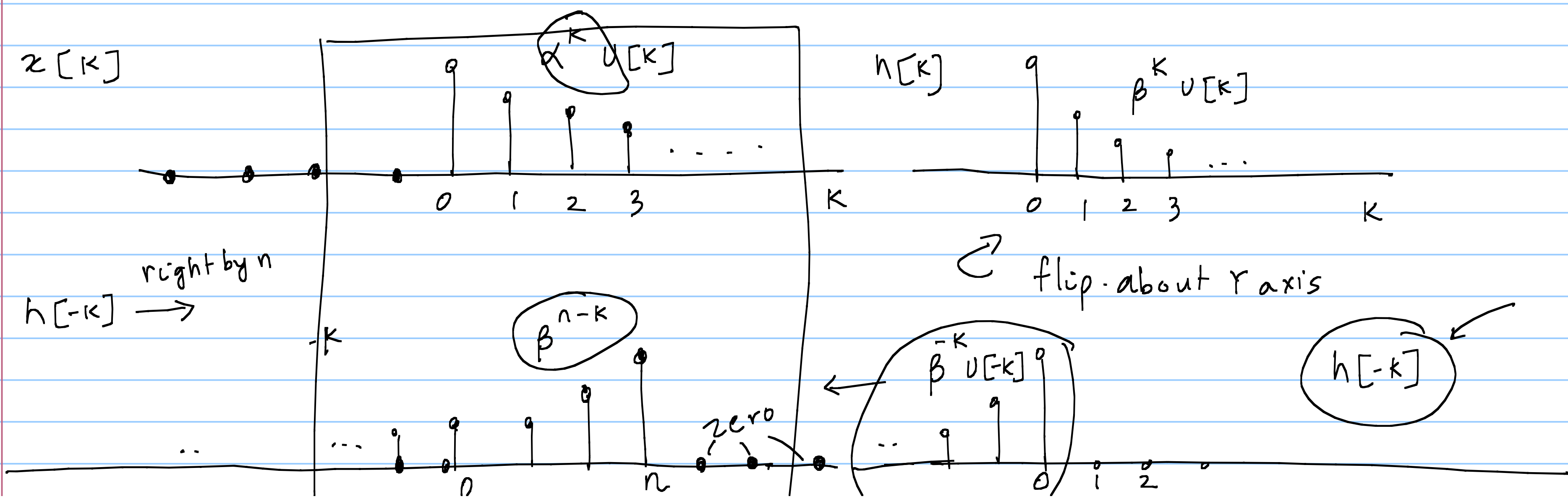
Example 1 : $x[n] = \alpha^n u[n]$ $h[n] = \beta^n u[n]$ α, β are two constants





$w_n[k] = x[k]\, h[n-k]$

k

For $n < 0$ $\quad w_n[k] = 0 \Rightarrow y[n] = 0$ for $n < 0$

For $n > 0$ $\quad w_n[k] = 0 \leftarrow k < 0$

$= \alpha^k \cdot \beta^{n-k} \quad 0 \le k \le n$

$= 0 \leftarrow k > n$

$$y[n] = \sum_{k=-\infty}^{\infty} w_n[k] = \sum_{0}^{n} \alpha^k \cdot \beta^{n-k} = \boxed{\beta^n \cdot \sum_{0}^{n} \left(\frac{\alpha}{\beta}\right)^k}$$

$$= \beta^n \cdot \frac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \frac{\alpha}{\beta}} = \frac{\cancel{\beta^n}}{\cancel{\beta^{n+1}}} \cdot \frac{\beta^{n+1} - \alpha^{n+1}}{\frac{\beta - \alpha}{\cancel{\beta}}} = \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}$$

$$\boxed{y[n] = \begin{cases} 0 & n < 0 \\ \dfrac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} & n \geq 0 \end{cases}}$$

(Just from Equations without pictures)

<u>Example</u> $x[n] = \alpha^n u[n]$ $h[n] = \beta^n u[n]$

$$y[n] = x[n] * h[n] = \boxed{\sum_{k=-\infty}^{\infty} x[k]\, h[n-k]}$$

$x[k] = \alpha^k u[k]$

$h[n-k] = \beta^{n-k} u[n-k]$

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] \cdot \beta^{n-k} u[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k \beta^{n-k}\, u[k]\, u[n-k]$$

$u[k]$ $u[n-k]$

$u[k] = 1 \quad k \geq 0$
$\quad\;\; = 0 \quad k < 0$

$$\boxed{\begin{aligned} u[n-k] &= 1 \quad n-k \geq 0 \\ &= 0 \quad n-k < 0 \end{aligned}}$$

$\Rightarrow \boxed{0 \leq k \leq n}$ $u[k]\, u[n-k] = 1$

For all other values of k $u[k]\, u[n-k] = 0$

$$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k \beta^{n-k} \underbrace{u[k]\, u[n-k]}$$

If $n < 0$

$$y[n] = 0$$

If $n \geq 0$

$$y[n] = \sum_{k=0}^{n} \alpha^k \cdot \beta^{n-k} \cdot 1 = \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k = \beta^n \cdot \frac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \frac{\alpha}{\beta}}$$

$$= \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}$$

$$\boxed{y[n] = \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha}\, u[n]}$$