Continuous-time Convolution Examples

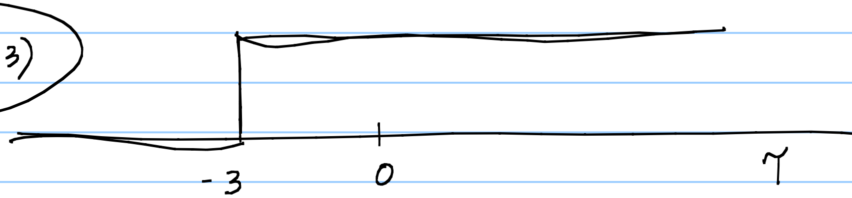
Note Title — 6/16/2011

1 $x(t) = u(t+3)$ $\qquad$ $x(\tau) = u(\tau+3)$

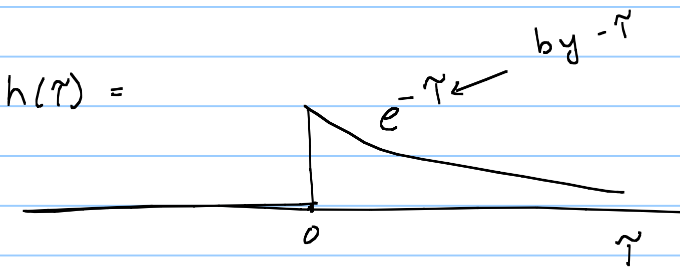
$h(t) = e^{-t}u(t)$ $\qquad$ $h(\tau) = e^{-\tau}u(\tau)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} \underbrace{x(\tau)\, h(t-\tau)}_{\boxed{w_{t}(\tau)}}\, d\tau$$

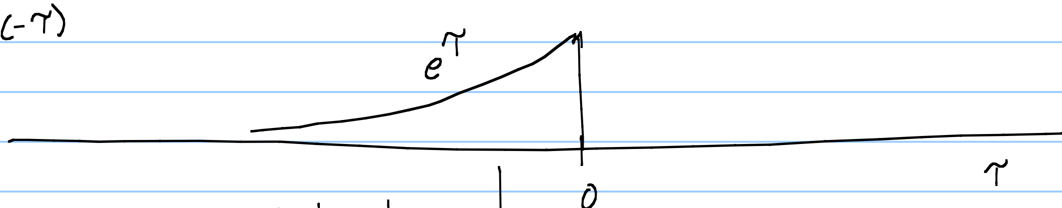
$x(\tau) = u(\tau+3)$ ✓



$h(\tau) =$

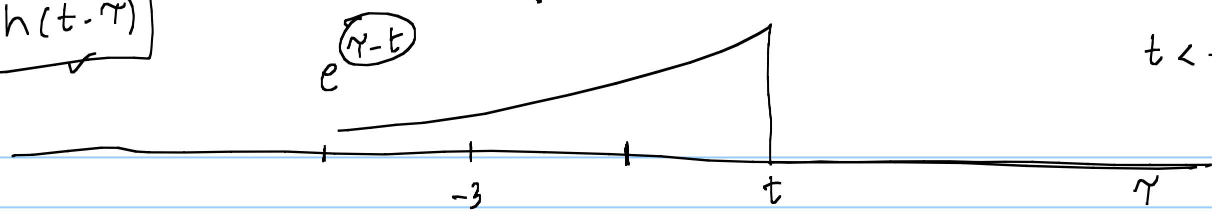


$h(-\tau)$

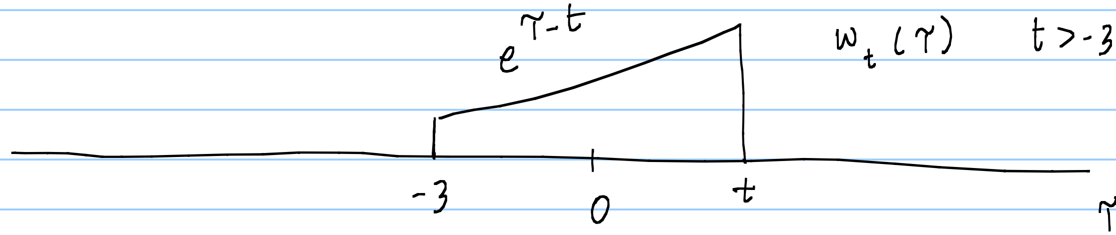


shift by t

$\boxed{h(t-\tau)}$



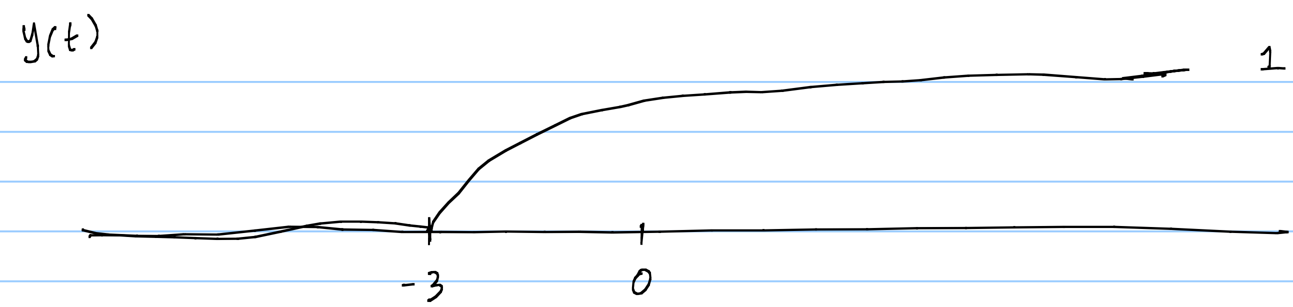
$t < -3 \implies \boxed{w_t(\tau) = 0} \quad -\infty < \tau < \infty$



$w_t(\tau) \quad t > -3$

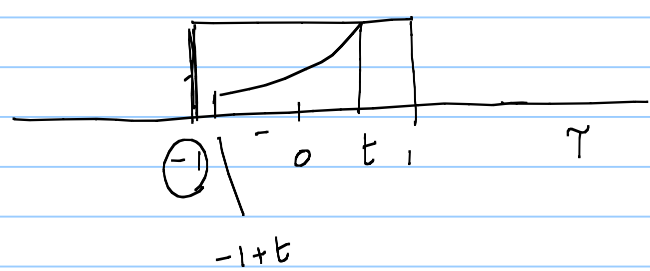
$$w_t(\tau) = e^{\tau - t} \quad \boxed{-3 \leq \tau \leq t}$$
$$= 0 \quad -\infty \leq \tau < -3$$
$$t < \tau < \infty$$

$$y(t) = \int_{-\infty}^{\infty} w_t(\tau)\, d\tau = 0 \qquad t < -3$$
$$= \int_{-3}^{t} e^{\tau - t}\, d\tau \qquad t \geq -3$$
$$= \left[e^{\tau - t} \right]_{-3}^{t} = 1 - e^{-3-t} \qquad t \geq -3$$

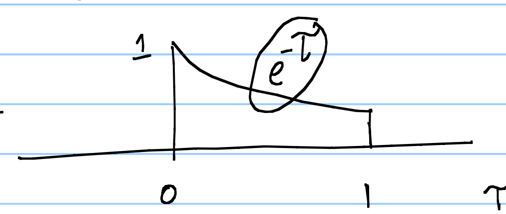


Example 2: $x(t) = u(t+1) - u(t-1)$ $\qquad$ $h(t) = e^{-t}\,(u(t) - u(t-1))$

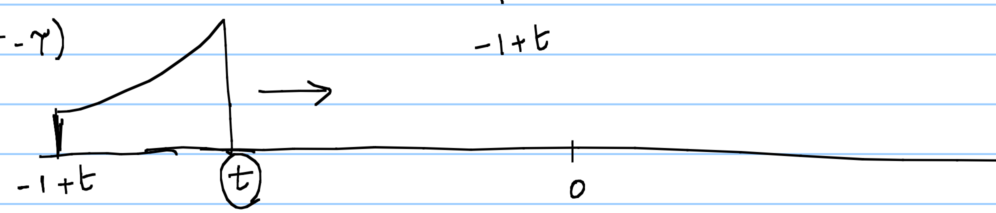
$x(\tau)$



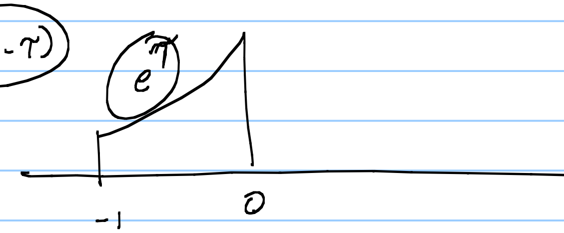
$h(\tau) =$



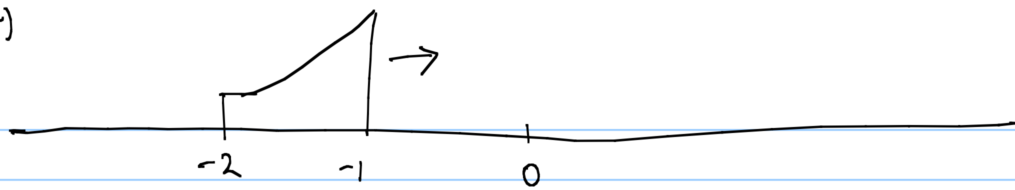
$h(t-\tau)$



$h(-\tau)$



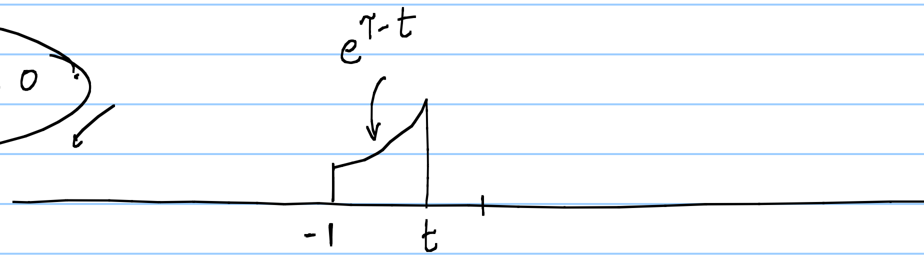
$h(t-\tau)$ $-1-\tau$



$w_t(\tau) = 0$ $\quad t < -1$

$$\boxed{y(t) = 0}$$

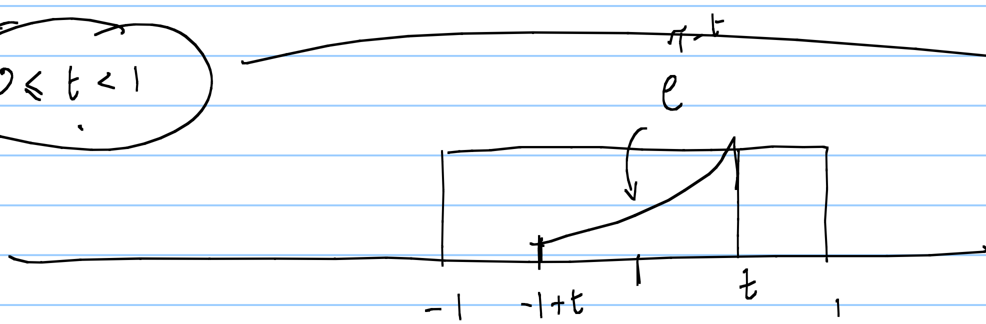
$-1 \le t < 0$



$w_t(\tau) = e^{\tau-t}$ $\quad -1 \le \tau \le t$

$$y(t) = \int_{-1}^{t} e^{\tau-t}\, d\tau = \left[e^{\tau-t}\right]_{-1}^{t} = \boxed{1 - e^{-1-t}}$$

$0 \le t < 1$

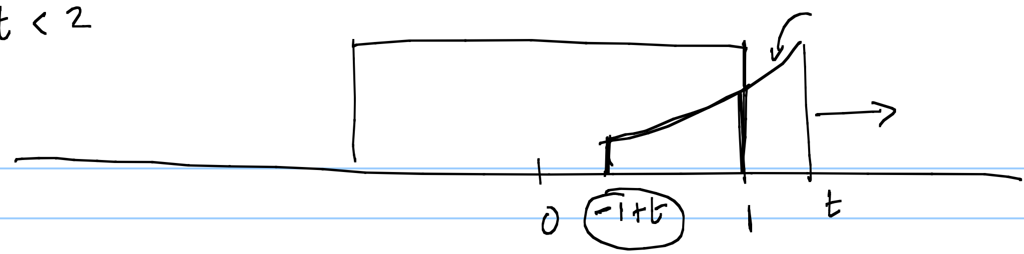


$w_t(\tau) = e^{\tau-t}$ $\quad -1+t \le \tau \le t$

$= 0$ $\quad$ other values of τ

$$y(t) = \int_{-1+t}^{t} e^{\tau-t}\, d\tau = \left[e^{\tau-t}\right]_{-1+t}^{t} = \boxed{1 - e^{-1}}$$

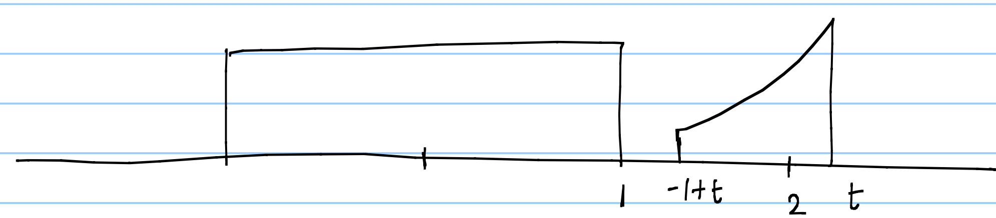
$1 \leq t < 2$



$$w_t(\tau) = e^{\tau - t} \qquad -1+t \leq \tau \leq 1$$

$$y(t) = \int_{-1+t}^{1} e^{\tau - t}\, d\tau = \left[e^{\tau - t}\right]_{-1+t}^{1} = \boxed{e^{1-t} - e^{-1}}$$

$2 \leq t$



$$w_t(\tau) = 0 \qquad -\infty < \tau \leq \infty$$

$$y(t) = 0$$

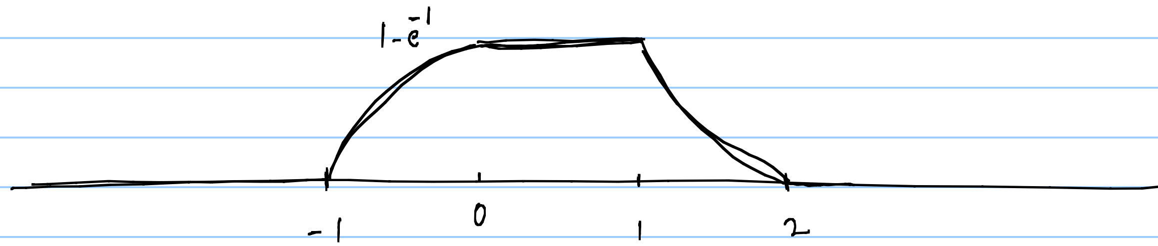
$$y(t) = \int_{-\infty}^{\infty} w_t(\tau)\, d\tau \qquad 0 \qquad t < -1$$

$$y(t) = 1 - e^{-1-t} \qquad -1 \leq t < 0$$

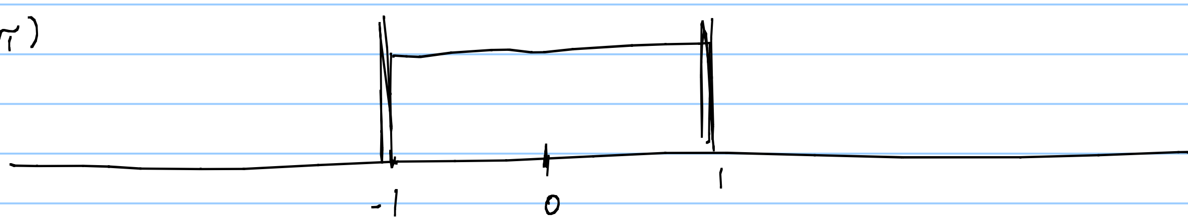
$$= 1 - e^{-1} \qquad 0 \leq t < 1$$

$$= e^{1-t} - e^{-1} \qquad 1 \leq t < 2$$

$$= 0 \qquad t > 2$$



$x(\tau)$



$h(\tau)$