Basic operations on Signals

Note Title 6/5/2011

$x(t)$

dependent ↑ independent variable

DT $x[n]$

Operations Performed on the dependent variable

* Amplitude Scaling: $y(t) = c\,x(t)$ $\qquad y[n] = c\,x[n]$

* Take two signals $x_1(t)$ and $x_2(t)$

$y(t) = x_1(t) + x_2(t)$ $\qquad x_1(t) = \cos(\omega_0 t)$

or

$= x_1(t) - x_2(t)$ $\qquad x_2(t) = t^2$

or

$= x_1(t) \cdot x_2(t)$ $\qquad y(t) = x_1(t) + x_2(t)$

or $\qquad = t^2 + \cos\omega_0 t$

$$y(t) = \frac{d\,x(t)}{dt} \quad \text{or} \quad y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

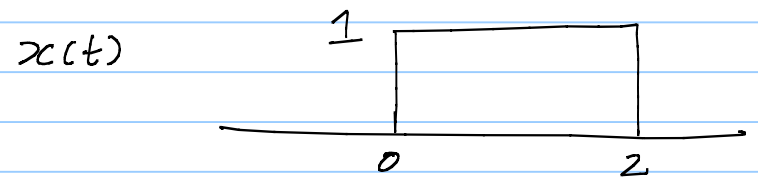
$x(t)$ $x(\tau)$

Operations Performed on the independent Variable

Time scaling:

$x(t)$ $\quad y(t) = x(at) \quad a > 1$

$y(t) = x(2t)$

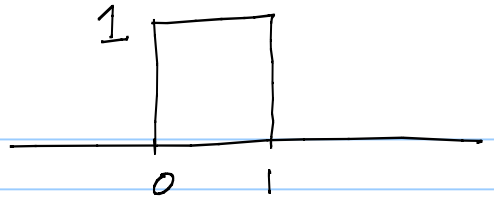


$y(0) = x(0)$

$y(\frac{1}{2}) = x(2 \cdot \frac{1}{2}) = x(1)$

$y(1) = x(2)$

$y(t)$



Suppose $a<1$

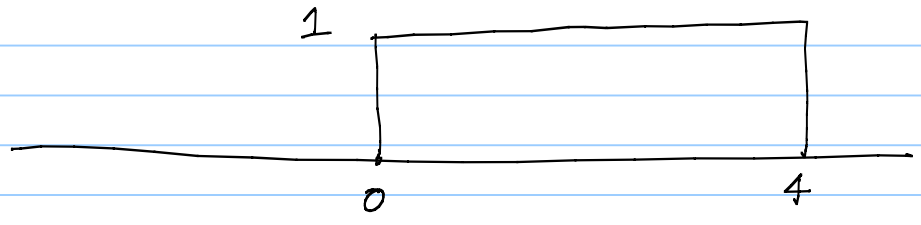
$a = \frac{1}{2}$

$y(0) = x(0)$

$y(1) = x(\frac{1}{2})$

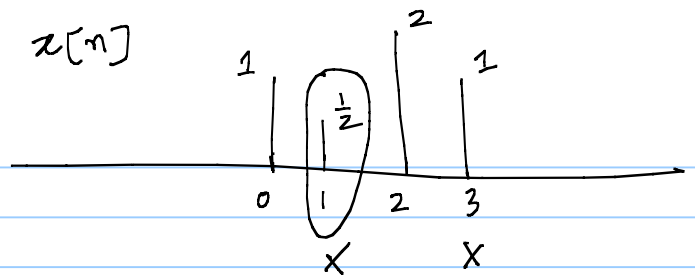
$y(2) = x(1)$

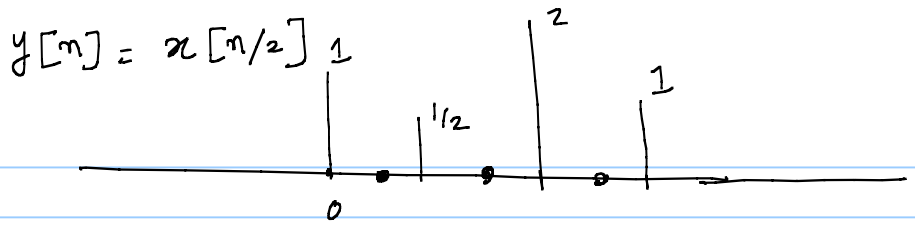
$\vdots$

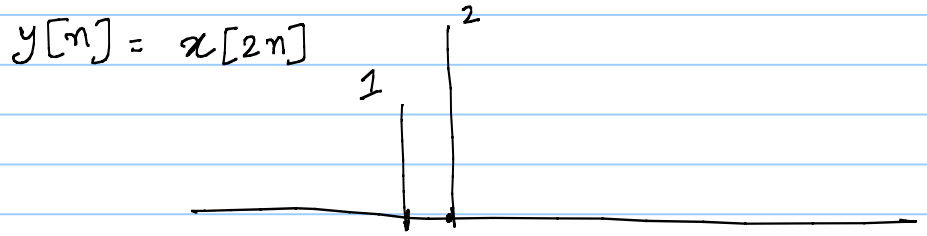


Suppose we have a DT signal $x[n]$

$$y[n] = x\left[\frac{n}{2}\right]$$

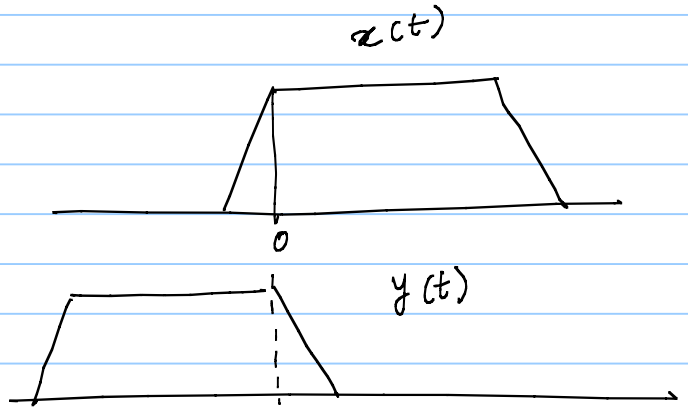






Reflection

$$y(t) = x(-t)$$



Time shifting

$y(t) = x(t - t_0)$

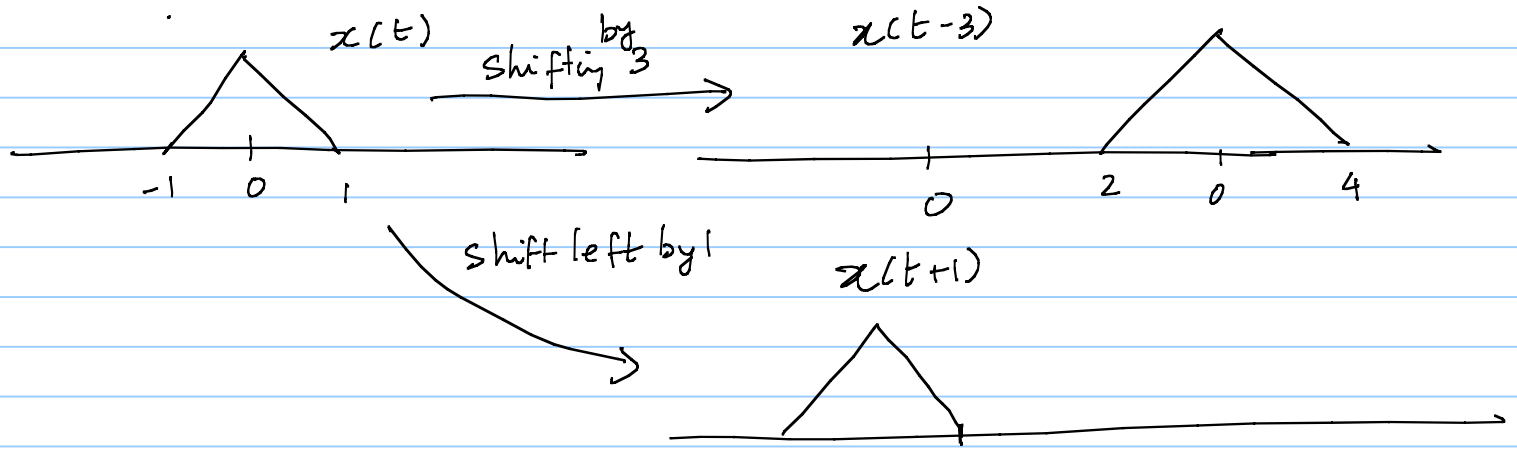
$y(0) = x(-t_0)$

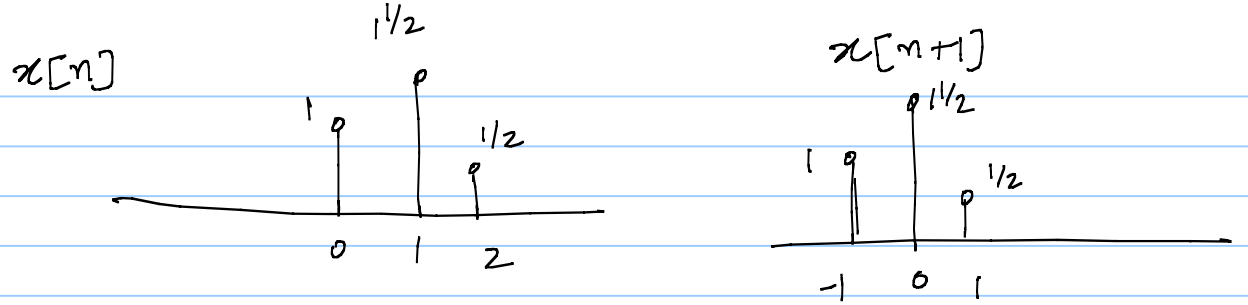
$y(1) = x(1 - t_0)$

$\vdots$

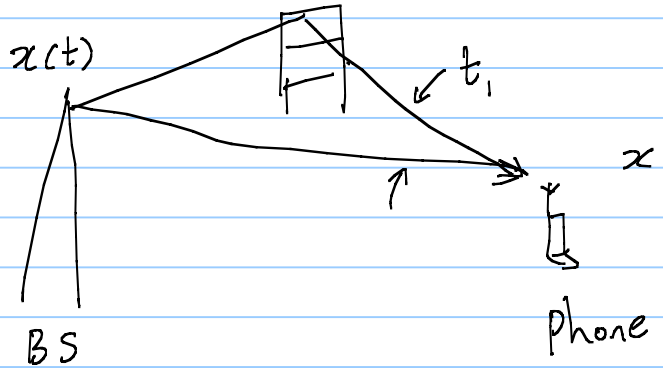
If $t_0 > 0$ then $x(t - t_0)$ is a right shift of $x(t)$

$t_0 < 0$ then $x(t - t_0)$ is a left shift





Example: Cellular Communication System



$x(t-t_0) + x(t-t_1)$

$y(t) =$

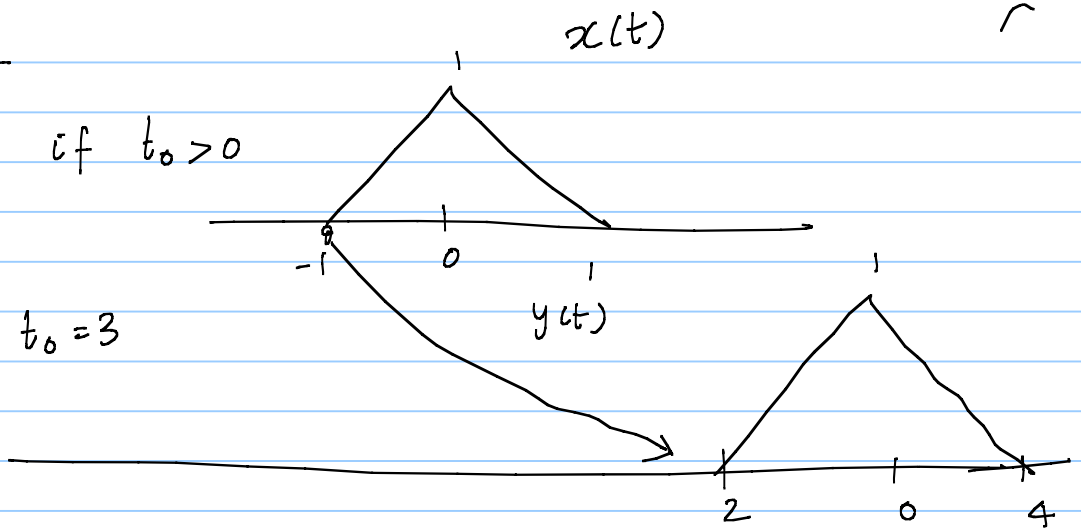
Time shifting

$y(t) = x(t - t_0)$ if $t_0 > 0$

$y(0) = x(-t_0)$

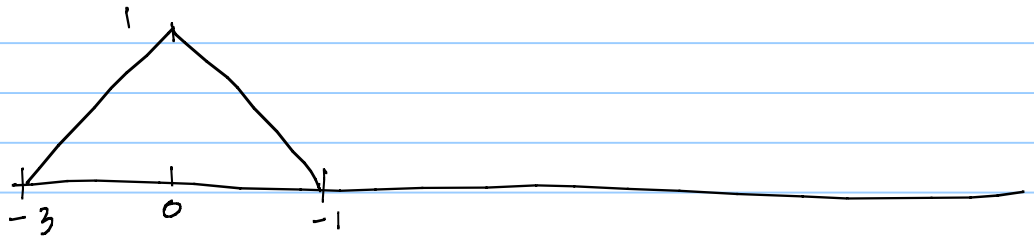
$y(1) = x(1 - t_0)$

$y(2) = x(2-3) = x(-1)$

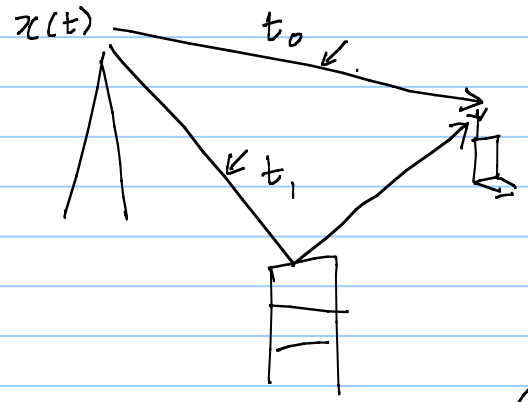


if $t_0 < 0$, then $y(t)$ is a shift of $x(t)$ to the left by t_0

$t_0 = -2$



Cellular Communication System



$$y(t) = x(t-t_0) + x(t-t_1) + \dots$$

$x[n]$ $\quad$ $y[n] = x[n-n_0]$ $\quad$ $n_0 > 0$ – shift right

$n_0 < 0$ – shift left

$x(t)$

$y(t) = x(at - b)$

$x(t) \xrightarrow[t \to t-b]{\text{shift right by } b} x(t-b) \xrightarrow[t \to at]{\text{Scale time axis by } a} x(at-b)$ ✓

$x(t) \xrightarrow[t \to at]{\text{Scale by } a} x(at) \xrightarrow[t \to t-\frac{b}{a}]{\text{If we shift by } \frac{b}{a}} x\left(a\left(t-\frac{b}{a}\right)\right) = x(at-b)$

$$y(t) = x\left(\frac{t-b}{a}\right)$$

$$x(t) \xrightarrow[t \to \frac{t}{a}]{\text{scale by } \frac{1}{a}} x\left(\frac{t}{a}\right) \xrightarrow[t \to t-b]{\text{Shift by } b} x\left(\frac{t-b}{a}\right)$$

$$x(t) \xrightarrow[t \to t-\frac{b}{a}]{\text{shift by } \frac{b}{a}} x\left(t-\frac{b}{a}\right) \xrightarrow[t \to \frac{t}{a}]{\text{scale by } \frac{1}{a}} x\left(\frac{t}{a}-\frac{b}{a}\right) = x\left(\frac{t-b}{a}\right)$$

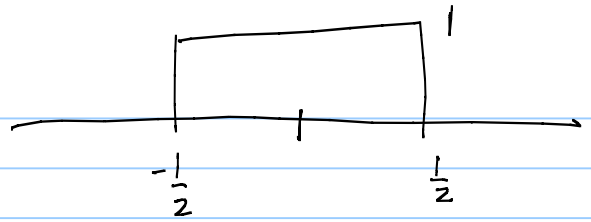
$x(-at-b)$ from $x(t)$

$x(t)$ —shift by b→ $x(t-b)$ —scale by a→ $x(at-b)$ —Reflect about Yaxis, $t \to t$→ $x(-at-b)$

$x(t)$ —Reflect, $t \to -t$→ $x(-t)$ —shift by b (left), $t \to t+b$→ $x(-(t-b))$ → $x(-t+b)$

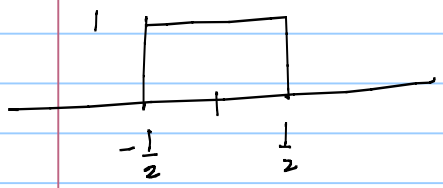
Be careful

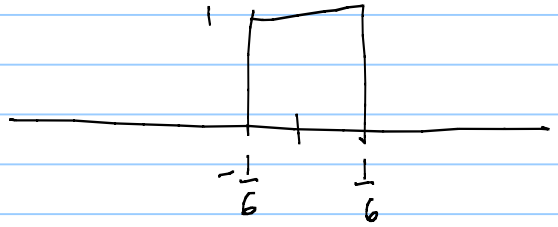
$x(t)$

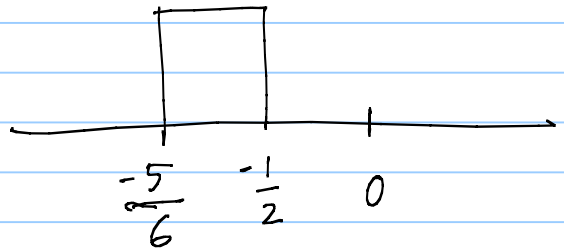


$y(t) = x(3t+2)$

$x(t)$ —scale by 3, $t \to 3t$→ $x(3t)$ —shift left by $\frac{2}{3}$, $t \to t+\frac{2}{3}$→ $x(3t+2)$

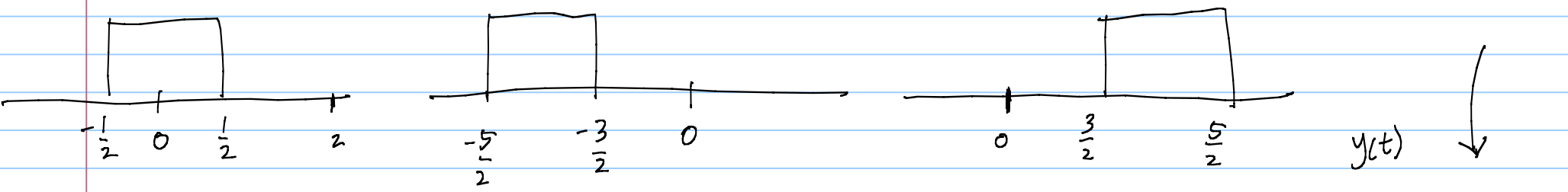


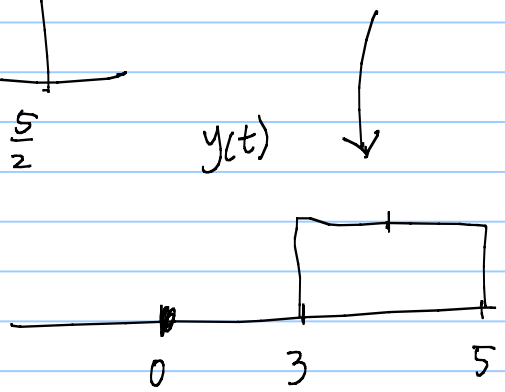




$$y(t) = x\left(2 - \frac{t}{2}\right) \leftarrow$$

$x(t)$ —shift left by 2→ $x(t+2)$ —reflect→ $x(-t+2)$ —Scale by 1/2→ $x\left(-\frac{1}{2}t+2\right)$

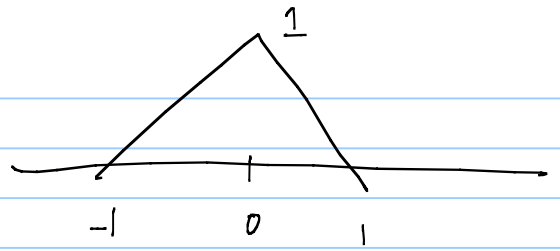




$t=0$ $\quad y(0) = x(2-0) = x(2) = 0$ ✓

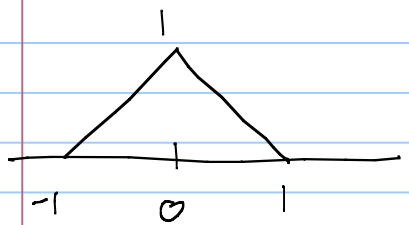
$\quad\quad\quad y(4) = x(2-2) = x(0) = 1$ ✓

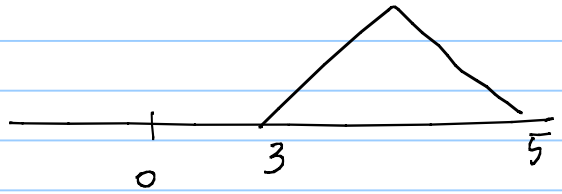
$x(t)$

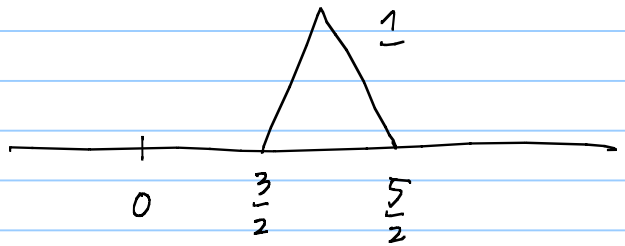


$$y(t) = x(2(t-2)) = x(2t-4)$$

$x(t)$ —shift rt by 4→ $x(t-4)$ —scale by 2→ $x(2t-4)$







$x(t)$ —scale by 2→ $x(2t)$ —shift by 2 to the right, $t \to t-2$→ $x(2(t-2))$